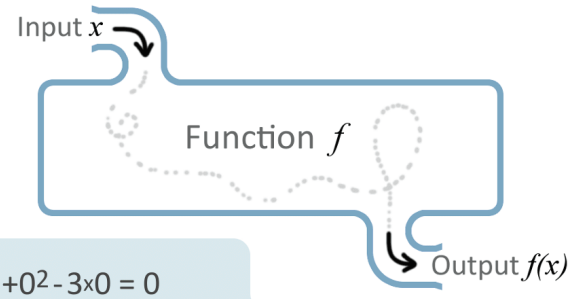
Functions & Graph Transformations

What is a Function?

A function describes a relation between two (or more) values. Each input value has one output.

For example: let $f(x) = x^3+x^2-3x$

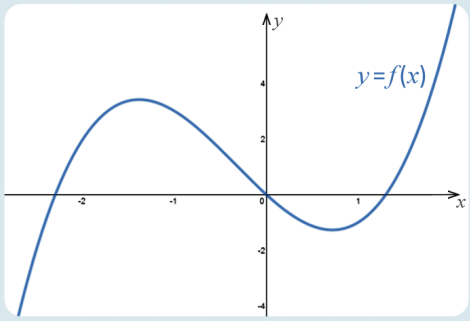
$f(2) = 2^3+2^2-3\times2 = 6, \quad f(0) = 0^3+0^2-3\times0 = 0$



A function is usually called f but can, like a variable, be called any letter or symbol.

Graphing a Function

A function is graphed the same as the "y = ..." equations you're used to, we just use "$f(x) = ...$" instead!



Transformations

Transformations cause functions to change in some way. Constants are used to either **translate** or **stretch** a function's graph.

Translate: a shift, the graph moves:

up & down	or	left & right
$f(x)+a$		$f(x+b)$

Stretch: (or squeeze) parallel to:

y axis	or	x axis
$cf(x)$		$f(dx)$

Top tip

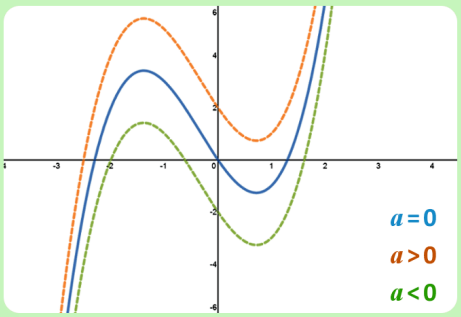
If the constant is **inside** the function, in with the x, then it causes transformations in the direction of the x axis.

If it is **outside** the function, operating on $f(x)$, then it causes changes in the direction of the y axis.

$f(x)+a$

Shifts the graph
up for $a>0$
down for $a<0$

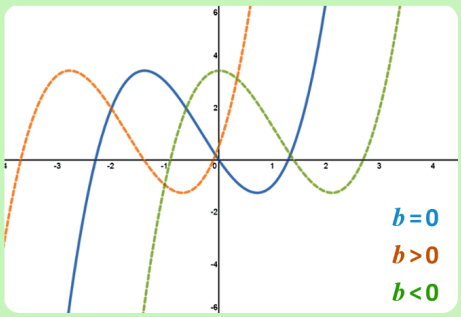
Think of moving the graph a units up the y axis



$f(x+b)$

Shifts the graph
left for $b>0$
right for $b<0$

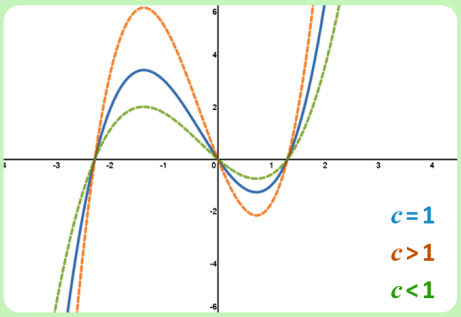
Think of moving the graph "$-b$" units along the x axis



$cf(x)$

Stretches the graph parallel to the y axis
out for $c>1$
(squeeze) **in for $c<1$**

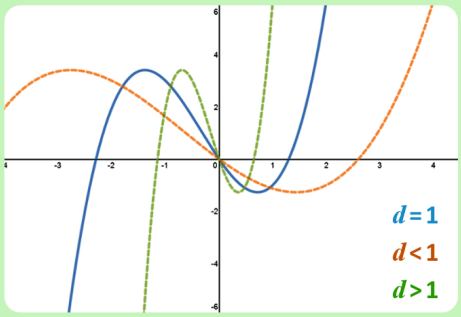
The scale factor is c, think of points being c times further from $y=0$



$f(dx)$

Stretches the graph parallel to the x axis
out for $d<1$
in for $d>1$

The scale factor is $1/d$, think of points being d times as close to $x=0$



These bottom two graphs only use positive constants. **Exercise:** can you sketch what they'd look like with negative constants?